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Gainesville State College
Thirteenth Annual Mathematics Tournament
April 14, 2007

Morning Component

Good morning!

Please do NOT open this booklet until given the signal to begin.

There are 40 multiple choice questions. Answer the questions on the electronic grading form by giving the best answer to each question.

The scoring will be done by giving one point for each question answered correctly and zero points for each question answered incorrectly or left blank. Thus, it is to your advantage to answer as many questions as possible, even if you have to guess. If there is a tie, question number 18 will be used again as a tie-breaker.

This test was designed to be a CHALLENGE. It is difficult, and you may not have time to complete all questions. Do not worry if you are unable to answer several of the questions. Instead, we hope that you will obtain satisfaction from those questions which you ARE able to answer.

You may write in the test booklet. You may keep your test booklet and any of your scrap papers. Only the electronic grading form will be collected and graded.

Good luck!

Do Not Open Until Signaled.

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Mathematics Tournament

You may write in this test booklet. Only the electronic form will be graded. Correct answers are awarded one point. Incorrect or blank answers are awarded 0 points.

3. Suppose f is differentiable on $(-\infty,\infty)$, $f(-1)=4$, and $\left|f'(x)\right|\le 1$

7. Evaluate $_{0}\ 3xe^{4x}\,dx$

10. Let f and g be differentiable functions such that $f'(x) = g(x)$ and $g'(x) = -f(x)$, and let $T(x) = [f(x)]^2 + [g(x)]^2$. Find $T'(x)$.

a) $T'(x) = 4f(x)g(x)$

b) $T'(x) = 2$

c) $T'(x) = 2f(x)g(x)$

d) $T'(x) = 0$

13. A tank filled with water is in the shape of an inverted cone 20 ft high with a circular base (on top) whose radius is 5 ft. Water is running out of the bottom of the cone at the constant rate of $2\ \text{ft}^3/\text{min}$. How fast is the level of water falling when the water is 8 ft deep?

a) $\frac{1}{\pi}$ ft/min

b) $\frac{1}{2\pi}$ ft/min

c) $\frac{1}{3\pi}$ ft/min

d) $\frac{1}{5\pi}$ ft/min

e) none of the above

14. Let $h(x) = f(g(x))$ where

16. Determine which of the following is <u>not</u> equal to the definite integral $\int_2^7 a x f(x) dx$.

a) $a\int_2^7 x f(x) dx$

b) $x\int_2^7 a f(x) dx$

c) $-\int_7^2 a x f(x) dx$

d) $\int_2^4 a x f(x) dx - \int_7^4 a x f(x) dx$

e) none of the above

17. A cylindrical gas tank with radius 3 ft and length 15 ft is buried 2 ft below ground level. The density of gasoline is 46 / ^{3}

19. Find the length of the curve $y = x^{2/3}$, $1 \le x \le 8$.

a) $\frac{8}{27}\left(80\sqrt{10} - 13\sqrt{13}\right)$

b) $\frac{1}{27}\left(80\sqrt{10} - 13\sqrt{13}\right)$

c) $\frac{8}{27}\left(13\sqrt{13} - 8\sqrt{10}\right)$

d) $\frac{1}{27}\left(8\sqrt{10} - 13\sqrt{13}\right)$

e) none of the above

20. Let $f(x)$ be a continuous function over the interval $(-\infty, \infty)$. Given $\int_a^b f(x)\,dx = 3$, $\int_c^d f(x)\,dx = -2$, $\int_b^d f(x)\,dx = 5$, where a, b, c, and d are real numbers with $a < b < c < d$. What is $\int_a^c f(x)\,dx$?

a) 12
b) 10
c) 8
d) 6
e) none of the above

21. Find the number of discontinuity points of the function

$$f(x) = \begin{cases} x+1 & \text{if } x \le 0 \\ \frac{1}{2} & \text{if } 0 < x \le 1 \\ 2-x & \text{if } 1 < x \le 2 \\ (2-x)^2 & \text{if } 2 < x \end{cases}$$

a) 0
b) 1
c) 2
d) 3
e) none of the above

22. Find the area of the region bounded by $y=\sqrt{9-x^2}$, $y=\sqrt{4-x^2}$ and $y=0$.

a) 5π

b) $\frac{5\pi}{2}$

c) $\frac{5\pi}{4}$

d) $\frac{5\pi}{9}$

e) none of the above

23. Find the average value of $f(x)=\begin{cases}\dfrac{x^2}{\sin x} & 0<|x|\le\dfrac{\pi}{2}\\ 0 & x=0\end{cases}$ over $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

a) 1

b) -1

c) $\frac{1}{\pi}$

d) $\frac{\pi}{4}$

e) none of the above

25. What value of x maximizes θ in the given figure:

a) $\sqrt{y^2 + y}$

b) $\sqrt{y^2 + yh}$

c) $\sqrt{y^2 + h}$

d) $\sqrt{y^2h + yh^2}$

e) none of the above

26. Suppose $f(0) = 0$, $f(1) = 1$, $f'(x) > 0$ for all x, and $f''(x) > 0$ for all x. Then

a) $(\ )$ $(\ )$

28. If $x = 0$ is a critical number of the function f, then what can be said about the function $g(x) \quad f(x \quad h) \quad k$,

31. Find the volume of the solid of revolution formed by revolving about the x-axis the region between the line $y = 0$ and $f(x) = e^{\frac{-x}{2}}\sqrt{\cos(x)+1}$, with $x \geq 0$.

a) $\frac{3}{2}\pi$

b) π

c) 3π

d) $\frac{1}{2}\pi$

e) none of the above

32. If f is twice differentiable and $\int_0^x f''(t)\,dt > 0$ for all x in $(0,1]$, then

a) f must be concave up on $[0,1]$

b) f must be increasing on $[0,1]$

c) f must be decreasing on $[0,1]$

d) f must be positive on $[0,1]$

e) none of the above

33. If f is continuous on $[-2, 3]$ with $f(-2) = 5$ and $f(3) = -4$, which of the following must be true?

I: $f'(x) = -\frac{9}{5}$ has a solution with $-2 < x < 3$

II: $-4 \leq f(x) \leq 5$ if $-2 < x < 3$

III: f attains a maximum value on $[-2, 3]$

a) I only

b) II onL,L,A,Ak$7(y

34. Suppose $f\left(x\right)=x^5+2x^3+7x-4$ and f^{-1} denotes the inverse of f.

Then $\left(\quad\right)$

37. The volume of a right circular cone is $V = \frac{2\sqrt{2}}{3}\pi$. Find the smallest possible total surface area of such a cone.

a) 4π

b) $2\pi\left(1+\sqrt{2}\right)$

c) 3π

d) $2\pi\sqrt{2}$

40. Find the area of the region bounded by the curves $f(x) = x + 1$ and $g(x) = x^2 - 2x + 1$.

a) $\frac{9}{2}$

b) 9

c) $\frac{3}{4}$

d) $\frac{5}{2}$

e) none of the above